The alternating-current-driven motion of dislocations in a weakly damped Frenkel–Kontorova lattice

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Abstract. By means of numerical simulations, we demonstrate that an alternating-current (ac) field can support stably moving collective nonlinear excitations in the form of dislocations (topological solitons, or kinks) in the Frenkel–Kontorova (FK) lattice with weak friction, as was qualitatively predicted by Bonilla and Malomed (Bonilla L L and Malomed B A 1991 *Phys. Rev. B* **43** 11 539). Direct generation of the moving dislocations turns out to be virtually impossible; however, they can be generated initially in the lattice subject to an auxiliary spatial modulation of the on-site potential strength. Gradually relaxing the modulation, we are able to get stable moving dislocations in the uniform FK lattice with periodic boundary conditions, provided that the driving frequency is close to the gap frequency of the linear excitations in the uniform lattice. The excitations that can be generated in this way have a large and noninteger index of commensurability with the lattice (so suggesting that the actual value of the commensurability index is irrational). The simulations reveal two different types of moving dislocation: broad ones, that extend, roughly, to half the full length of the periodic lattice (in that sense, they cannot be called solitons); and localized soliton-like dislocations, that can be found in an excited state, demonstrating strong persistent internal vibrations. The minimum (threshold) amplitude of the driving force necessary to support the travelling excitation is found as a function of the friction coefficient. Its extrapolation suggests that the threshold does not vanish at zero friction, which may be explained by radiation losses. The moving dislocation can be observed experimentally in an array of coupled small Josephson junctions in the form of an *inverse Josephson effect*, i.e., a direct-current-voltage response to the uniformly applied ac bias current.

1. Introduction

The role of solitons (localized nonlinear collective excitations) in nonlinear dynamical models of solid-state and polymer physics is well known (see, e.g., [1, 2]). In real systems, the most crucial problem is compensation of dissipative losses. In particular, the losses always give rise to a friction force acting on a moving soliton. The friction is usually balanced by a driving force, which may be induced by an external field. If the losses are weak enough, i.e., the corresponding model is *underdamped* (which is typical for the Josephson junctions [3]), the necessary driving field is also weak. In uniform continuous systems with friction, the external drive cannot support progressive (on average) soliton motion unless the field contains a dc component. However, an ac drive with *zero* dc component can give rise, in the presence of friction, to motion of a soliton at a nonzero mean velocity in a continuous system periodically modulated in space [4], or in a discrete lattice. A physically relevant modulated continuous

model, in which this effect was studied in detail analytically and numerically, describes a long Josephson junction with a periodically modulated critical current density (which can easily be realized in the experiment by means of periodic modulation of the thickness of the junction's dielectric layer [5]):

$$\phi_{tt} - \phi_{xx} + [1 + \epsilon \sin(x/L)] \sin \phi = -\alpha \phi_t + \gamma \sin(\omega t) \tag{1}$$

where ϕ is the magnetic flux inside the junction, the coordinate x and time t are measured in units of the Josephson penetration length and inverse gap frequency of the junction, α is the dissipative constant, and ϵ and L are the amplitude and period of the modulation, γ and ω being the amplitude and frequency of the ac bias current driving the junction. A soliton in the model (1) moving at a nonzero mean velocity represents an *inverse* Josephson effect.

The basic mechanism, which is common to the modulated continuous model (1) and the discrete models described below, is that the motion of a topological soliton (kink) with a nonzero mean velocity v may be possible, in the lossy system, in a *resonant* case, when the period $2\pi/\omega$ of the driving ac force is a multiple of the time L/v which the soliton takes to pass one spatial period: $2\pi/\omega = mL/v$, where $m = 1, 2, 3, \ldots$ is the order of the resonance. More general subharmonic resonances, with

$$v = (m/M)(L\omega/2\pi) \tag{2}$$

(m and M being mutually prime integers), are possible too.

In the lattices, a similar effect was predicted in models of two different kinds: Toda-type chains, in which the interaction between neighbouring particles is anharmonic (with no substrate potential); and lattices of the Frenkel–Kontorova (FK) type, combining the harmonic interaction between the neighbours and an anharmonic on-site potential. Note that the FK lattice finds its straightforward (and, as a matter of fact, most realistic) physical realization in the form of a an array of coupled small Josephson junctions (traditionally, the FK model was applied to crystal-lattice dynamics [2], where, however, this model may be too idealized because of the complexity and non-one-dimensionality of the real crystals, strong dissipation and thermal fluctuations, etc). Recently, precise experiments with the Josephson arrays have begun [6,7]. The necessary drive is provided by the ac bias current uniformly applied to all the junctions. Then, the progressive motion of the soliton-like excitation can be easily observed in the form of mean dc voltage across the junctions.

In [8], the ac-driven motion of a dislocation was predicted analytically for the Toda-lattice models, and in [9] it was observed in direct numerical simulations of an ac-driven weakly damped Toda lattice (in its so-called dual form). Recently, a similar effect was described analytically and found numerically also in a *parametrically* ac-driven weakly damped Toda lattice [10].

In the earlier work [11], the existence of a moving kink (lattice dislocation) was predicted for the ac-driven FK model with weak friction (cf. equation (1)),

$$\ddot{\xi}_n - a^{-2}(\xi_{n-1} + \xi_{n+1} - 2\xi_n) + \sin \xi_n = -\alpha \dot{\xi}_n + \gamma \sin(\omega t) \tag{3}$$

where $\xi_n(t)$ is the coordinate of the nth particle in the lattice, and the inverse coupling constant a plays the role of the lattice spacing in the quasicontinuum limit. Like in the continuum model (1), in equation (3) time is rescaled so that the coefficient in front of the on-site nonlinear term $\sin \xi_n$ is $\equiv 1$. In the lattice, it is natural to measure the travelled distance by counting the number of the sites that the soliton has passed. The accordingly defined velocity u of the ac-driven soliton in the lattice was predicted in [11] to be, in the general case,

$$u = (m/M)(\omega/2\pi) \tag{4}$$

where, like in equation (2), m and M are the super- and subharmonic resonance orders. In view of the evident reflectional symmetry of the lattice, the resonant velocity may have either

sign. In a physical or numerical experiment, the sign of the velocity is determined by the initial conditions. Note also that the resonant velocity (4) is not related to the phase or group velocity of the free excitations in the ideal (undamped and undriven) linearized FK lattice.

Along with the resonant velocity, an important characteristic of the ac-driven motion is the threshold value $\gamma_{\text{thr}}(\alpha, \omega)$ of the drive's amplitude, i.e., a minimum value of the amplitude which, at fixed values of α, ω, and other parameters, admits the ac-driven motion. In the modulated continuous model (1), the threshold was found analytically in the framework of the perturbation theory, treating ϵ, α, and γ as small parameters, and taking the kink solution to the unperturbed sine–Gordon (SG) equation as the zero-order approximation. This analytical result was demonstrated to be in good agreement with direct simulations [4]. The perturbation theory was also applied to the different versions of the ac-driven damped Toda lattice, which had produced a prediction for the threshold amplitude [8, 10] that was in a reasonable agreement with the numerical simulations reported in [9, 10].

For the ac-driven damped FK lattice, the situation is more difficult, as even the unperturbed FK lattice, i.e., the one without the friction and drive terms, is far from being solvable in any sense. Numerical simulations, starting from the work [12], have demonstrated that the unperturbed FK lattice does not support any fully stable moving kink. Instead, the discreteness gives rise to emission of radiation by the moving kink (which may produce strong resonant effects [13]), i.e., an effective radiation friction force. Moreover, in certain cases the emission of radiation may produce strong resonant effects [13]. In any case, the ac drive in the model (3) must compensate both the direct dissipative losses and the radiation.

Because no well-defined zero-order approximation for the moving soliton is available, the only practical possibility for getting analytical estimates is to use the quasicontinuum approximation, which was done in [11] in order to predict the threshold for the existence of the ac-driven soliton in the ac-driven damped FK lattice. However, the quasicontinuum approximation had produced a threshold amplitude that was exponentially large in the discreteness parameter a (see equation (3)). In reality, the system may easily develop an instability and slide into a chaotic regime before reaching such a large value of the driving amplitude (see, e.g., [14]). Therefore, without direct numerical simulations, it is not clear whether one can rely upon the predictions of the quasicontinuum approximation.

The first simulations of the kink's dynamics in the model (3) were reported in [14], where no case of the ac-driven kink's motion was found, and it had been concluded that this regime is impossible. In contrast with that, a recent work [15] reported other results of simulations of a FK lattice that consisted of 20 particles with $a = 1$: the ac-drive period $T \equiv 2\pi/\omega$ took the values 50 or 25, while the dissipative constant was $\alpha = 0.1$. Note that the corresponding friction-braking time $\sim 1/\alpha$ is significantly smaller than the period T; hence this regime was, in fact, *overdamped*. The most significant effect observed in [15] was the depinning of a kink trapped in the lattice under the action of a rather strong ac drive. In most cases, the overdamped motion of the depinned kink was a diffusive drift; however, at some values of the driving force, a nonzero mean velocity predicted by equation (2) with $m = M = 1$ was observed. Thus, the results reported in [15] confirm, in a limited case, the qualitative prediction made in [11]. However, the basic characteristic of the ac-driven motion, namely, its threshold, was not considered in [15], and, in fact, the most interesting *weakly* damped case was not dealt with at all in that work.

Our objective is to present results of systematic numerical simulations that confirm the existence of the stably moving dislocation-like collective excitations in the weakly damped ac-driven FK lattice with periodic boundary conditions (b.c.), that include a phase jump of 2π, necessary to support the topological soliton. These collective excitations are found in two very different forms: broad ones, that extend, roughly, to half the full length of the periodic lattice (so

the term 'soliton' is not appropriate for them) and propagate keeping a virtually constant shape; and narrow (really localized) quasi-solitons that are found in an *excited* form, demonstrating persistent internal vibrations: the 2π-kink periodically splits into two π-subkinks that then recombine back into it.

Because of the lack of a good initial guess, it proved to be virtually impossible to generate these excitations directly. Therefore, it was necessary to devise a special indirect procedure that made it possible to generate the moving collective excitations, which is described in section 2. In sections 3 and 4, we summarize the numerical results obtained, respectively, for the broad and narrow moving dislocations. The threshold characteristics for both types of dislocation are presented in section 5, and the paper is concluded by section 6.

2. The mode of the simulations

All the attempts to *directly* find the dynamical regime sought for in the weakly damped model (3) have failed. A clue to a successful quest was to start from the case studied in the numerical part of the work [4]. Indeed, the simulations of the modulated continuous model (1) performed in that work amounted to a numerical solution of a *discrete* system with periodic modulation,

$$\ddot{\xi}_n - a^{-2}(\xi_{n+1} + \xi_{n-1} - 2\xi_n) + [1 + \epsilon \sin(2\pi n/N)] \sin \xi_n = -\alpha \dot{\xi}_n + \gamma \sin(\omega t) \qquad (5)$$

(cf. equations (3)), subject to the periodic b.c. with the phase shift of 2π, $\xi_N \equiv \xi_0 + 2\pi$. Note that the modulation period introduced in equation (5) exactly coincides with the full size of the lattice.

It was easy to catch the moving 2π-kink (dislocation) in simulations of the modulated discrete system (5). The initial displacements $\xi_n(t = 0)$ and the velocities $\dot{\xi}_n(t = 0)$ were taken as per the exact kink solution of the unmodulated continuum SG equation, moving at the velocity

$$u_{\mathrm{in}} = N\omega/2\pi. \qquad (6)$$

This velocity is suggested by the condition of the fundamental resonance between the kink's motion and ac drive, which was our starting point. Actually, the possibility of launching the moving dislocation in the modulated system was not sensitive to details of the initial configuration, i.e., the driven dislocation is a sufficiently strong attractor in the weakly damped modulated system. Then, the simulations were continued, gradually decreasing the modulation amplitude ϵ until it vanished. In some cases, the ac-driven regime persisted up to $\epsilon = 0$. The ac-driven regime of the motion of the dislocation in the unmodulated lattice was regarded as stable if it persisted for $\geqslant 10\,000$ circulations around the periodic lattice (the corresponding time is larger than that sufficient for a freely moving dislocation to be stopped by the friction force, typically by a factor $\gtrsim 500$).

The study of the transition between the ac-driven regimes in the modulated FK lattice (3) and the uniform one (5) is of interest in itself. Deferring a detailed consideration of this issue to another work, we here demonstrate its most salient feature: keeping all the parameters but the modulation amplitude ϵ constant, and gradually decreasing ϵ, we observed an abrupt decrease of the velocity u by a jump at a well-defined finite value of ϵ (figure 1). This feature proved to be quite generic for the transition from the ac-driven motion in the modulated lattice to the motion in the uniform one.

3. The broad dislocations

The first noteworthy numerical result for the uniform lattice ($\epsilon = 0$) is that a minimum value of the lattice size N, at which it was possible to extend the ac-driven regime up to $\epsilon = 0$, was
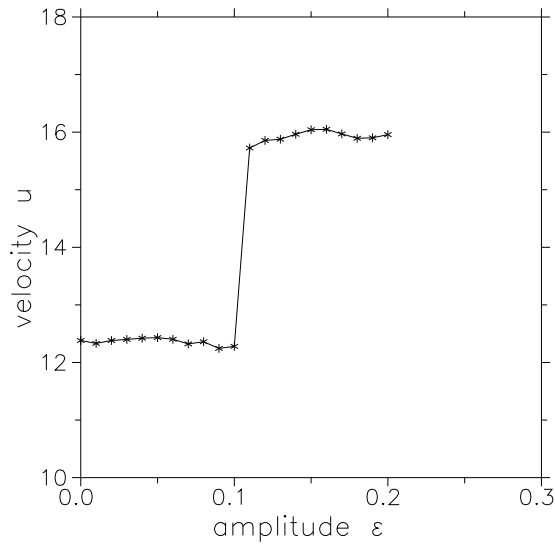
Figure 1. A typical example of the abrupt decrease of the ac-driven soliton's velocity in the modulated FK lattice with the decrease of the modulation amplitude ϵ. In this figure, $\gamma = 0.5$, $a = 0.0564$, $\alpha = 0.005$, and $N = 100$. In this and all the other figures, $\omega = 1$.

$N_{\mathrm{min}} = 12$, while no upper limit for N has been found (at least, up to $N = 100$). Another noteworthy peculiarity is that the ac-driven motion of the dislocation could not be supported in the uniform FK lattice, unless the driving frequency ω belonged to a narrow interval of width $\Delta\omega \sim 0.03$ around the value $\omega = 1$. Therefore, all the plots displayed below pertain to $\omega = 1$.

A possible explanation of the latter feature can be provided by a *cascading mechanism* described below. First, we note that $\omega = 1$ is exactly the *gap frequency* in the spectrum of the linear excitations, $\xi_n \sim \exp(\mathrm{i}kn - \mathrm{i}\chi t)$, in the uniform FK model (3), which is

$$\chi^2(k) = 1 + 4a^{-2}\sin^2(k/2). \tag{7}$$

The spatially uniform ac drive with the frequency ω, applied to the lattice, excites uniform oscillations in it, which is the first step of the cascading. At the next step, the modulational instability induced by the nonlinearity stimulates decay of the uniform oscillations into a pair of travelling waves with the same frequency ω and the wavenumbers $\pm k$ related to ω by the dispersion relation (7). Finally, at the third step, the travelling wave can support (*drag*) a moving dislocation. Note that dragging a dislocation (kink) by a given periodic travelling wave was earlier considered in terms of other physical models (see, e.g., [16]). If ω is close to the gap frequency of the lattice, the generated travelling waves have a large wavelength $2\pi/k \approx 2\pi/a\sqrt{\omega - 1}$ (provided that $\omega > 1$), and a relatively large amplitude $\sim[(\omega^2 - 1)^2 + \alpha^2]^{-1/2}$. This may be an explanation for the fact that the moving dislocations supported by the ac drive have a large size (see below), and that they can be supported only when ω is sufficiently close to 1.

In the established regime obtained at $\epsilon = 0$ by means of the procedure described above, its *commensurability index* r was found from the numerical data, using equation (4) as the definition:

$$r \equiv 2\pi u/\omega \tag{8}$$

where u is realized as the mean velocity of the ac-driven soliton. The value of r thus found was always quite high, but smaller than its large initial integer value $r_{\mathrm{in}} \equiv N$ in the modulated

lattice (see equation (6)), which is explained by the abrupt drop of u seen in figure 1. Moreover, the value of r found at $\epsilon = 0$ turns out to be *non*integer. In figure 2, we display the plot of r versus a, obtained at fixed values $N = 25$, $\omega = 1$, $\gamma \simeq 0.5$, and $\alpha = 0.005$. It is necessary to stress that, at $N = 25$, it was not possible to find the stable ac-driven regime at $\epsilon = 0$ outside the limited range of the values of a shown in figure 2.
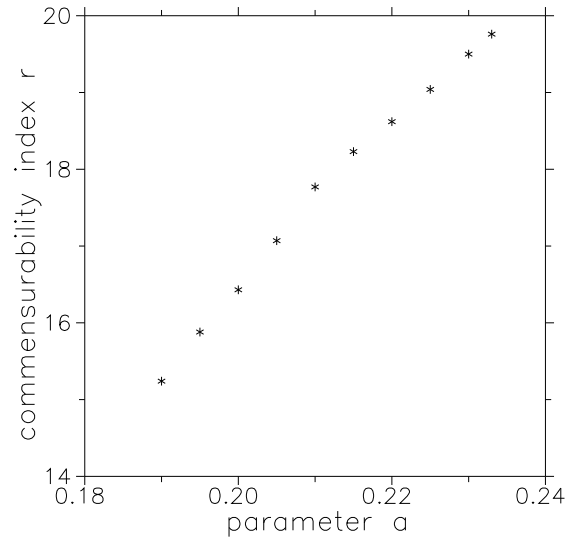

Figure 2. The effective commensurability index defined as per equation (8) versus the inverse harmonic coupling constant a, as obtained from the numerical data. Because $\omega = 1$, this plot, in fact, also shows the velocity of the moving dislocation. The other parameters are $\gamma \simeq 0.5$, $\alpha = 0.005$, and $N = 25$.

At other values of the parameters N, γ, and α, the dependencies $r(a)$ were found to be quite similar to those shown in figure 2. In particular, for each value of the lattice size N, there is its own interval of the values of a, in which the ac-driven motion can be observed. Defining $\bar{a}$ as the central point of this interval, we were able to make the following observation: the product $\bar{a}N$ changes only from 5.25 to 5.28 for N varying between 25 and 100. A straightforward implication of this observation is that, in the quasicontinuum limit, the length $L \equiv aN$ of the closed system admitting the ac-driven motion of the dislocation is nearly constant (recall, however, that the ac-driven motion of the dislocation (kink) is impossible in the uniform continuous system).

Actually, the commensurability index is *irrational*, which is seen from a typical phase portrait of the ac-driven regime displayed in figure 3(a). To generate the phase portrait, an arbitrary lattice site was selected, and on its phase plane $(\xi_n, \dot{\xi}_n)$, a discrete trajectory was plotted, consisting of the points picked up at $t = (2\pi/\omega)j$, where j is discrete time taking integer values. The quasicontinuous evolution of ξ_n obvious in figure 3(a) implies that the commensurability index is indeed irrational. Note, however, that the phase portraits show no trace of dynamical chaos. Simultaneously, they strongly suggest that, on the phase plane $(\xi_n, \dot{\xi}_n)$, there must exist two *fixed points*, one being a saddle corresponding to the self-intersection of the trajectory, the other one being a central point inside the loop formed by the self-intersecting trajectory. The fixed points correspond to $r = N$, i.e., the solution in which the soliton completes exactly one round trip in the periodic lattice during one period of the external drive. However, we have not been able to capture the corresponding solutions
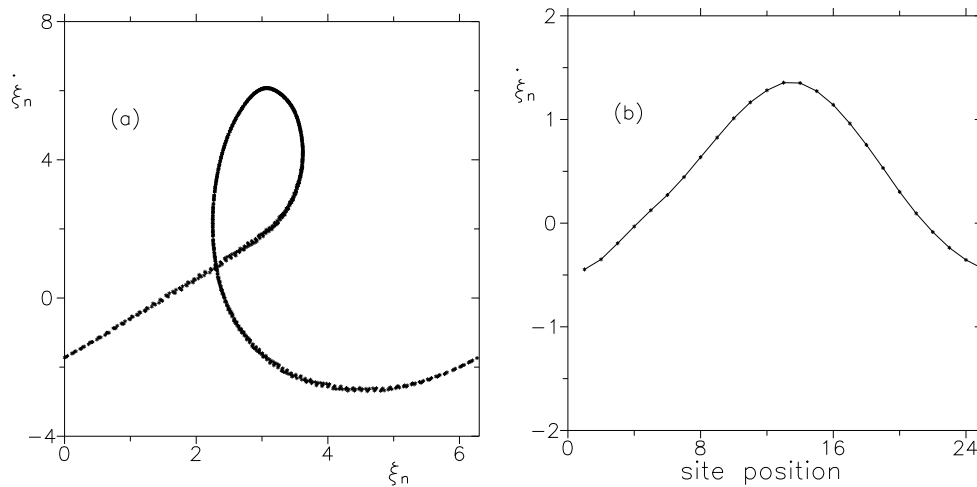
Figure 3. Typical examples of the phase portrait (a) and the shape (b) of the broad ac-driven dislocation at $\gamma = 0.5$, $a = 0.226$, $\alpha = 0.005$, and $N = 25$. The mean velocity of this dislocation is $u = 3.10$.

in the simulations. It is obvious that the saddle is an unstable fixed point; hence it cannot be found in the dynamical simulations. As regards the central fixed point, it would be stable in a conservative system, but it may easily become unstable in our dissipative driven model, which is a plausible explanation for the impossibility of capturing this fixed point.

Although the solution of the present type actually found is nonstationary, its nonstationarity is mild: the dislocation does not manifest conspicuous internal vibrations, and, at any moment of time, its shape is close to that in figure 3(b), which is an instantaneous snapshot showing $\dot{\xi}_n$ versus the lattice site number n, taken at an arbitrarily chosen moment of time. Note that the presentation of the shape in this form is relevant for the arrays of coupled small Josephson junctions [7]: $\dot{\xi}_n$ is the instantaneous voltage at the nth junction.

As one sees from figure 3(b), the moving excitation is broad, extending, roughly, to half the full length of the lattice. Note, however, that this excitation is, definitely, a dislocation, as it bears the topological charge 2π. In view of the fact that this moving dislocation is observed at the values of the discreteness parameter $a \simeq 0.2$ (see figure 2), and the size of the dislocation is $\gtrsim 10$ (figure 3(b)), one may conjecture that the dislocation should have its counterpart in the corresponding continuous model, i.e., the ac-driven damped SG equation. However, we recall once again that the progressive motion of a collective excitation is impossible in the ac-driven continuous systems. Alternatively, one can assume that the moving dislocation is a 2π-kink massively 'dressed' by the long-wavelength lattice phonons, which would comply with the above-mentioned cascading mechanism, that may feasibly account for the energy transfer from the ac drive to the dislocation. Much more work is necessary to elaborate this possibility.

4. The narrow dislocations and their vibrations

The broad dislocations considered in the previous section can hardly be called 'solitons'. However, a dislocation of another type, which is much better localized, can also be found. To this end, we start from the modulated FK model (5), with the periodic b.c. corresponding to the lattice size $2N$ (instead of the size N dealt with above), so that the initial dislocation, being

essentially the same as in the previous section, is twice as narrow in comparison with the full lattice size. Then, the procedure of decreasing the modulation amplitude ϵ to zero produces a nondecaying excitation essentially different from that considered in the previous section. As well as the broad dislocations, the new ones were observed only in a narrow interval of the frequencies $\Delta\omega \simeq 0.03$ around $\omega = 1$.

A typical phase portrait of the dislocation of this type is shown in figure 4, which is obviously different from figure 3(a). The moving dislocation demonstrates strong persistent internal vibrations. Approximately periodically, it oscillates between two configurations, one of which corresponds to a clearly localized 2π-kink, and the other to a set of two separated π-subkinks (figure 5). We stress that the internal vibrations of the narrow dislocation (as well as its progressive motion) continue indefinitely, being, obviously, permanently supported by the ac drive. A numerically calculated temporal Fourier transform of the solutions (not shown here) demonstrates that the frequency of the internal vibrations of the narrow dislocation moving at the mean velocity u is close to $a \cdot u$. It is necessary to mention that the kink in the unperturbed FK lattice is known to have an *internal vibrational mode* [2, 17], which may be quite a natural explanation for the persistent internal oscillations of the narrow dislocation. The internal mode can be easily excited by the periodic perturbation exerted by the lattice on the dislocation moving past it. Of course, the progressive motion of the dislocation may be significantly affected by the interaction of the corresponding degree of freedom with the internal vibrations. In this work, we do not consider the latter issue in detail.
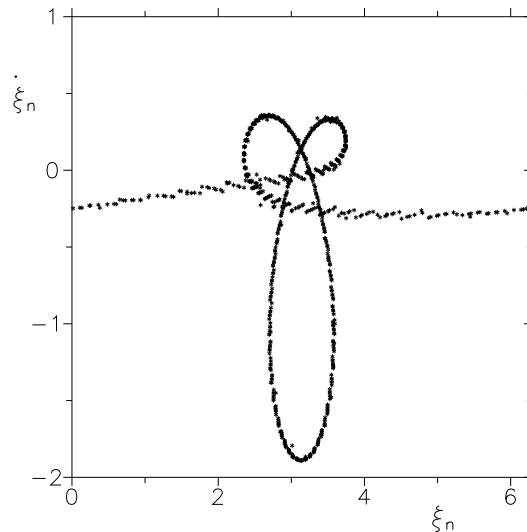

Figure 4. A typical example of the phase portrait of the narrow ac-driven dislocation at $\gamma = 0.5$, $a = 0.226$, $\alpha = 0.005$, and $2N = 50$. The mean velocity of this dislocation is $u = 2.80$.

Thus, this type of moving collective excitation can be easily generated only in a vibrating state. On the other hand, the fixed points corresponding to the centres of the three loops seen in figure 4 (two upper and one lower) should represent this dislocation in its ground state. Though it is virtually impossible to directly generate the dislocation in this state, its shape can be effectively restored. To this end, we picked up a number of points belonging to one of the three loops and distributed uniformly along it. A presumably stationary shape of the narrow dislocation in its ground state was produced by juxtaposing and averaging the configurations taken at the selected points belonging to the loop. The result is shown in figure 6. As one
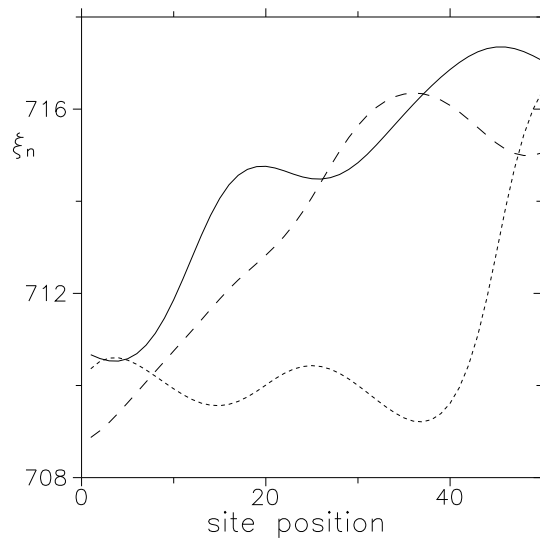
Figure 5. Three configurations between which the narrow dislocation vibrates: a 2π-kink (short-dashed line), the two separated π-subkinks (solid line), and an intermediate profile (long-dashed line). The values of the parameters are the same as in figure 4, except the actual size of the lattice which, according to the procedure adopted for the generation of the narrow dislocations, is $2N = 50$.
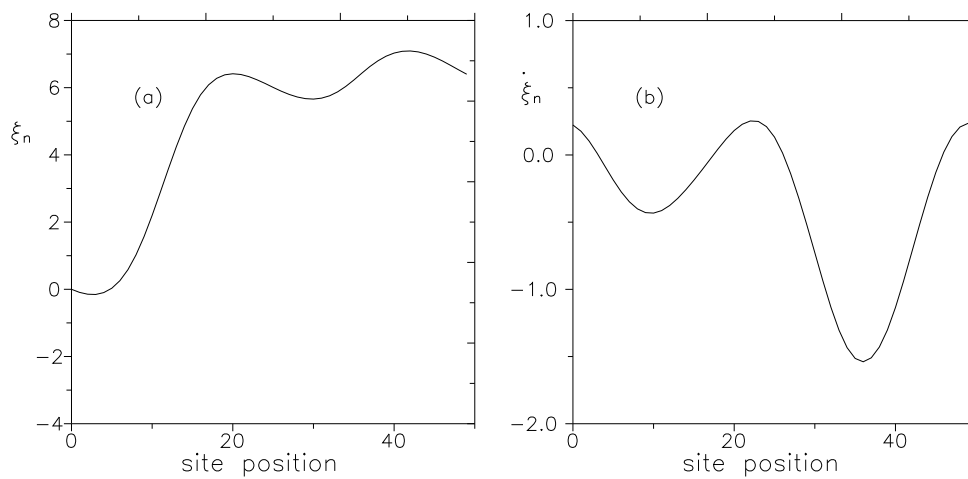

Figure 6. The indirectly retrieved configuration of the narrow dislocation in its ground state: (a) ξ_n versus the discrete coordinate n; (b) $\dot\xi_n$ versus n. The values of the parameters are the same as in figure 5.

sees in this figure, the ground state is indeed a narrow soliton-like dislocation, whose width is about a quarter of the full lattice size. The 'tail' attached to this narrow dislocation may, in principle, be either an artifact generated by the approximation used, or a genuine feature produced by the long-wavelength lattice phonons dressing the dislocation. We stress that the juxtaposing and averaging procedure has produced, with a reasonable accuracy, the *same* result when applied to all three loops in figure 4. Because the above-mentioned 'tail' was reproduced

virtually in the same form in all three cases, we conjecture that it is a genuine feature of the narrow dislocation, rather than an artifact. Accurate examination of the internal pulsations of the narrow dislocation shows that the pulsations are quite irregular (probably, chaotic). They have, in an approximate sense, a sort of basic period (not quite constant), which is close to six periods of the ac drive. Thus, there is no obvious resonance between the internal oscillations of the narrow dislocation and the external drive. Instead, this seems like a typical portrait of chaotic oscillations in a weakly damped nonlinear dynamical system driven by a periodic external force (as is well known, the dynamical regime in such a system should be, generally, chaotic, even if the system has a single degree of freedom).

A natural question is whether more types of dislocation can be produced by a generalization of the procedure that gave rise to the narrow dislocation, starting with the modulated lattice of the full size $3N$, $4N$, etc. The answer to this question is *negative*: at least, in the case of the full sizes $3N$ and $4N$, no new dislocation could be generated. So we just have the cases N and $2N$; this is too little for us to infer an infinite limit. The fundamental question of whether the dislocation that we have investigated is a soliton or not cannot be answered satisfactorily with the simulation presented here. However, in the two cases investigated, the head of the excitation is still interacting with the tail, so we should for the moment conclude that it is not a soliton.

5. The threshold characteristics

As was mentioned above, a basic characteristic of the ac-driven regime is its threshold, i.e., a minimum value γ_{thr} of the drive's amplitude γ that allows one to support the motion of the dislocation at a nonzero mean velocity. Because the driving force must (first of all) compensate the friction, we present, in figure 7, γ_{thr} versus the friction constant α at the driving frequency
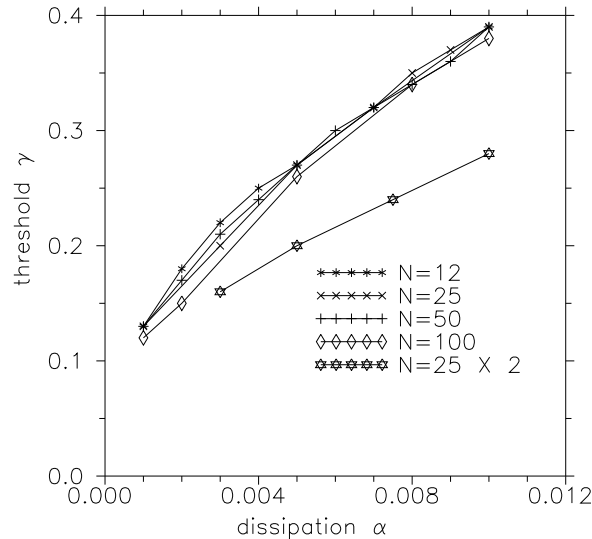

Figure 7. The dependencies $\gamma_{thr}(\alpha)$, the value of a being selected so that the soliton's velocity $v \equiv au$, defined as per the quasicontinuum approximation, keeps a constant value, $v = 0.7$. The stars, crosses, vertical scores, and rhombuses pertain, respectively, to the broad dislocations in the lattices with the lengths $N = 12$, $N = 25$, $N = 50$, and $N = 100$, respectively. The David's shields represent the narrow dislocations in the lattice with $2N = 50$.

$\omega = 1$ and different values of the lattice size N, selecting the values of the inverse coupling a so as to have a fixed value of the soliton's velocity $v = au$, defined as per the quasicontinuum approximation, in each plot. The data are presented for both types of dislocation, broad and narrow. A remarkable feature is that the plots $\gamma_{\text{thr}}(\alpha)$ for the dislocations of the former type are nearly universal, depending very weakly upon the lattice size N (while the size of the broad dislocation strongly depends on N, being $\simeq N/2$ according to the above results). It is also noteworthy that, for the narrow dislocations, the threshold is significantly lower, which may be related to the fact that, on average, this dislocation involves in the collective motion fewer particles in the lattice, and hence the dissipative losses are smaller.

A straightforward perturbation theory predicts the dependence $\gamma_{\text{thr}}(\alpha)$ to be linear [4, 8, 11]. The dependencies displayed in figure 7 are, in fact, not far from being linear, but an extrapolation (with a first-order-polynomial best fit) suggests that γ_{thr} remains different from zero at $\alpha = 0$. A natural explanation for this is that the driving force must compensate not only the direct dissipative losses, but also additional losses induced by the emission of radiation [13]. The internal mode can be easily excited by the periodic perturbation exerted by the lattice on the dislocation moving past it. Of course, the progressive motion of the dislocation may be significantly affected by the interaction of the corresponding degree of freedom with the internal vibrations. In this work, we do not consider the latter issue in detail.

Thus, at very small α, the radiation losses may dominate, demanding a finite drive's amplitude in the limit $\alpha \to 0$. Indeed, at extremely low values of the damping (α below ≈ 0.001, the lowest damping shown in figure 7), the radiation processes seem to dominate in the simulations. Actually, the ac-driven motion of the dislocation is very difficult to capture in the case of extremely weak dissipation, i.e., the dissipation is a stabilizing factor for the ac-driven dislocation (as long as the system does not become overdamped).

6. Conclusions

In this work we have demonstrated, by means of direct simulations, that two species of moving collective nonlinear excitation of the dislocation type (distinguished by the topological charge 2π) may exist in the ac-driven weakly damped FK lattice with periodic boundary conditions. Direct generation of the moving solitons turned out to be impossible; however, they can be generated initially in the lattice subject to a spatial modulation of the on-site potential strength. Then, gradually decreasing the modulation depth, we were able to find stable moving dislocations in the uniform FK lattice, provided that the lattice size is $N \geqslant 12$, and the driving frequency is close to the gap frequency of the linearized model.

All the collective excitations that were found have a large noninteger index of commensurability with the lattice. The results suggest that the exact value of the commensurability index is likely to be irrational. Moving dislocations of two different types have been found. The first type represents broad dislocations with a nearly stationary shape, extending to approximately half of the whole length of the lattice. The other type is represented by the collective excitations that, when generated directly, demonstrate strong persistent internal vibrations between the configurations corresponding, respectively, to a relatively narrow 2π-kink and two well-separated π-kinks. A shape of the dislocation of the latter type in the ground (nonvibrating) state was recovered indirectly, by means of juxtaposing and averaging many configurations picked up from a loop surrounding the corresponding fixed point. This ground state proves to be a soliton-like 2π-kink whose width is about a quarter of the lattice size.

The threshold amplitude of the driving force was found as a function of the friction constant α. An extrapolation of this dependence suggests that a nonzero drive amplitude remains necessary at $\alpha \to 0$, which may be explained by radiation losses. However, the

ac-driven dislocation becomes virtually unstable at vanishingly small values of α.

The effects studied in this work theoretically can be easily observed in an array of linearly coupled small Josephson junctions, in the form of a dc voltage generated by the ac bias current uniformly applied to the array. Note that, once the shape of the moving dislocation in the uniform lattice has been found, the corresponding initial configuration can easily be generated in the experiment; hence there is no real need to invoke the auxiliary spatial modulation of the lattice, which was a crucial trick in the simulations.

Although qualitative explanations of some features of the observed effects were put forward in this work, full understanding of the ac-driven motion of dislocations in weakly damped nonlinear lattices is still lacking.

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